

# Staggered Symmetric Passing by Sean Gandini

## Prechac Transformation

The Prechac transformation is a clever way of transforming solo siteswaps into numerous fascinating passing patterns. In fact every siteswap can be transformed into many different passing patterns. The resulting patterns will bear a structural resemblance to the siteswap they emanated from.

We call these patterns symmetric because each juggler does the same sequence of selfs and passes. In addition to being symmetric, the patterns are also staggered, by this we mean, that the jugglers do the same thing but at different times.

This system is a great way of generating all the possible passing patterns within given restrictions, as well as providing a useful intuitive understanding of passing patterns to people already familiar with siteswap notation. The idea behind the system was originally described in an article by French Juggler and mathematician Cristophe Prechac which was posted on rec.juggling in 1999.

Essentially the system takes a siteswap of  $n$  objects and of period  $p$  and transforms it into a passing pattern for 2 jugglers. It does this **by adding or subtracting half of the period to any of the throw values in the siteswap, and transforming this into a pass.**

### Notation:

The patterns are notated using a simple extension of siteswaps which involves putting a  $p$  next of a pass. This is essentially Jack Boyce's passing notation. The notation describes each passer individually regardless of how many other jugglers are playing. That way a self will be written as a 3 whether its a solo pattern or in a 7 person pattern.

Since most of the patterns in this article are symmetric, with all the jugglers doing the same job, in many situations only one juggler's job is shown. So for example the well known 7 object Triple Singles, if written in its full form:

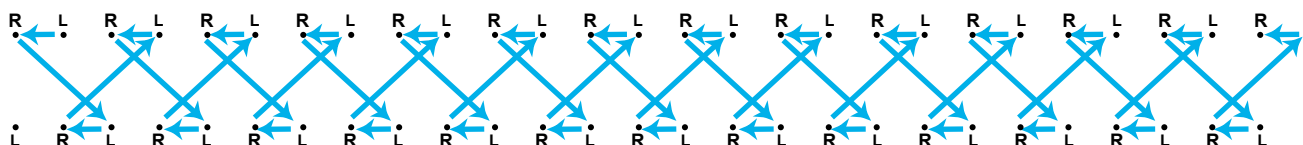
$\langle 5p\ 3\ 3\ 3 \mid 3\ 3\ 5p\ 3 \rangle$

The first part of the pattern gives Juggler 1's role and the second part gives juggler 2s role.

So for conciseness we will often only write:  $5p\ 3\ 3\ 3$ .

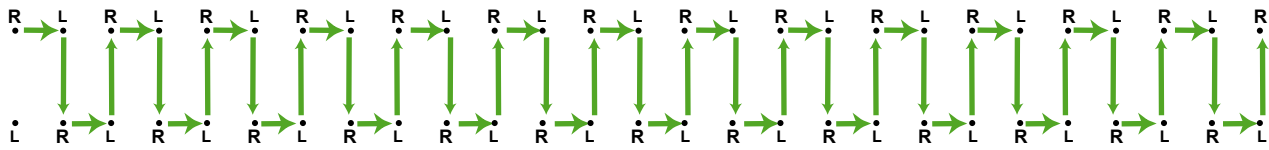
### Simple Examples of the Prechac Transformation

Lets look at some examples of this. Lets take the siteswap 31 the two ball shower. This is a period 2 pattern. We can add half the period to either of the digits. If we add half the period to the first digit in the siteswap we get  $4p1$ , a 5 object passing pattern.



A relatively easy and intuitive 5 object 2 count juggling pattern that has the same feeling as the 2 ball shower, 31.

On the other hand if we add half the period to the other digit we get 32p



An interesting 2 count where the selves are bigger than the passes.

Other examples:

33 transforms into 4p3 the seven object two count.

333 334.5p transforms into the 7 Object 3 count

Notice that when we add half the period we end up with a passing pattern with  $2n + 1$  objects and when we subtract half the period we get a pattern with  $2n - 1$  objects, where  $n$  is the amount of objects in the original pattern. This implies that a one pass patterns will have an odd amount of objects. It is therefore not possible, for example to create a 6 object one pass pattern. In fact it is not possible to create a one pass pattern with an even amount of objects.

## Prechac's Symmetric Patterns categorised

One of the most common problems, when encountering the myriad of new patterns that Prechac's system generates, is how does one quickly know how to juggle them. If you are an experienced passer, intuition will help you, however it can take a while to know whom to pass to and at what time. This was the reason i attempted to classify these patterns.

The system actually came into being from trying to do some of the 3 person patterns which appeared on the popcorn charts which in terms of list and theory are the precursors of these patterns; we spent an afternoon trying to juggle some 10 club triangles and it took us a very long time to work out how to juggle a handful of patterns. Once the patterns got going they were reasonably easy, but it was frustrating to have to spend more time working a pattern out than actually juggling it.

So initially i grouped 3 person patterns into a few manageable categories, that way one could use pattern templates to quickly know how to juggle any new pattern.

Once i had classified the 3 person i realised that one could do the same thing with 2 person patterns and indeed any amount.

Below is the classification system that we ended up with and that has been useful for the last few years.

## 2 jugglers

We can divide the passes into four basic categories:

### 1) **Classic Passes**

Classic passes are the most frequently juggled passing patterns. They are even period patterns which in the face to face position can be juggled with the all passes going from right hands to left hands. They are named classic because some of the most popular classic patterns predate this system and indeed notation in general. I would hasten to guess that most of the jugglers reading this have already juggled a classic pattern. The 7 club two count 4p3 is a classic pattern, as is the 7 club 4 count 5p333.

**Classic passes are generated when an Odd digit is transformed upwards or downwards within an Even Period siteswap.**

On our lists they will be coloured [blue](#).

### Example patterns

3 Objects:

[3p](#) 1 1 1 A fun three object four count pattern.

5 Objects:

[4p](#) 1 One hand throws one hand zaps.

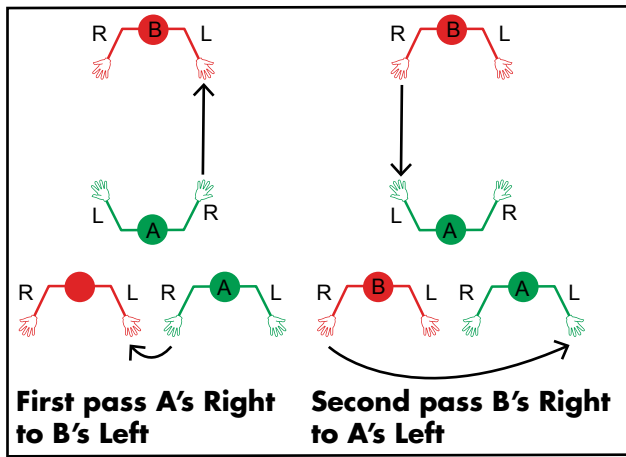
7 Objects:

[4p](#) 3 The classic seven two count:  
Done with clubs as double passes and single selfs.

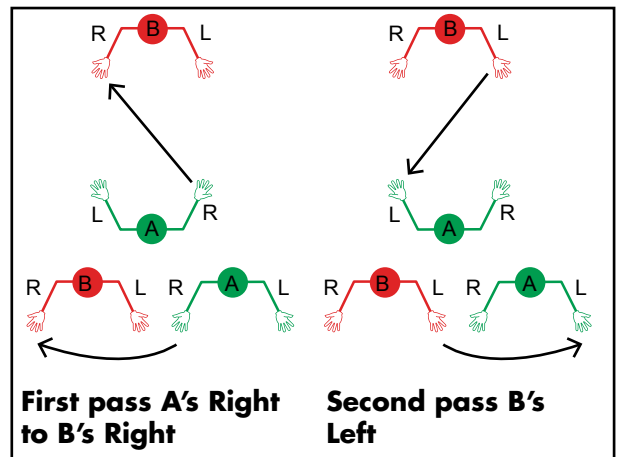
[5p](#) 3 3 3 Seven object four count:  
Seven 4 count or "triple singles"

As we mentioned before the simplest way of juggling classic patterns is face to face, with the right hands throwing to the left hands. However if you wish to make your life more complex or indeed for many fun side to side patterns there are actually 4 different ways of juggling these patterns:

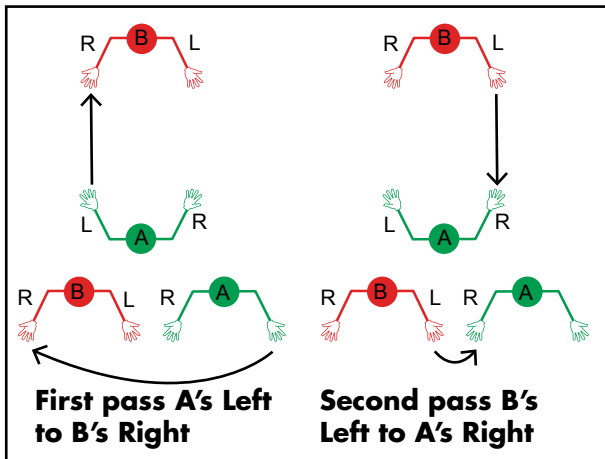
We call these 4 different ways **hand circulations**. We say there are 4 **hand circulations** for a classic pass. The hand circulation of a pattern tells us, for each individual juggler, which hand throws (right or left) and which hand catches (right or left). So in this case there are 4 different possible ways of doing this pass. Note that the hand circulation is not the same as the juggler position. The table below shows the 4 possible circulations in 2 different positions. Note also that the second two circulations are mirror images of the first two.



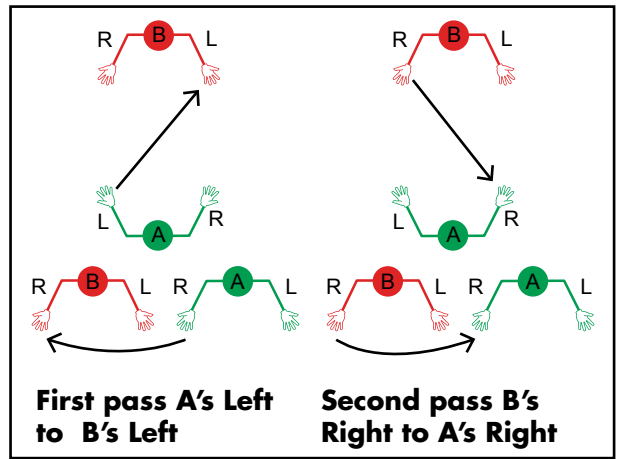
Circulation 1



Circulation 2



Circulation 3



Circulation 4

## 2) **Equihanded** Passes

Equihanded passes are even period passing patterns. What makes them particular is that the hand that passes is the hand that receives the pass. This means that only one of the juggler's hands is involved in passing and catching.

The name Equihanded comes from the Latin word equi meaning equal, the same. It is a word we invented and we affectionately shortened to equi. As far as we are aware equi patterns emanated from the notation and there were no popular equihanded patterns predating the notation.

Equihanded passes are generated when an even digit is transformed upwards or downwards within an even period siteswap.

On our lists they will be coloured **red**.

### **Example patterns**

3 Objects

**4p** 0 1 1

One could think of this as a slow share of a 3 ball cascade interrupted by a couple of zaps.

5 Objects

**4p** 4 1 1

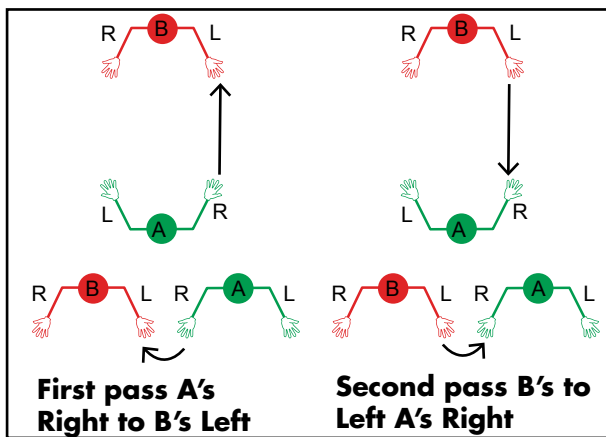
The pattern above with the gap filled in!

7 objects

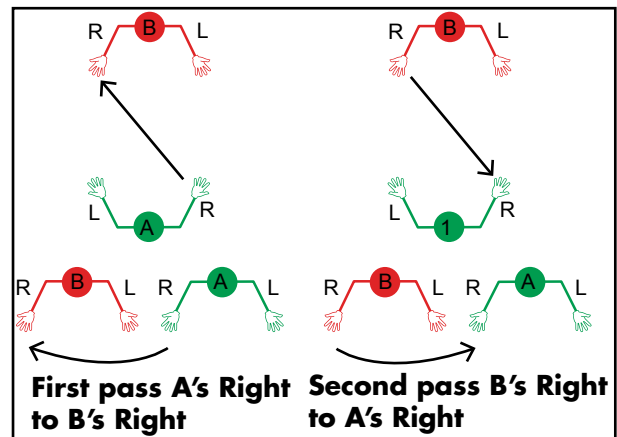
4 3p

A cheeky pattern where one hand continuously passes and catches whilst the other hand does 2 objects in one hand.

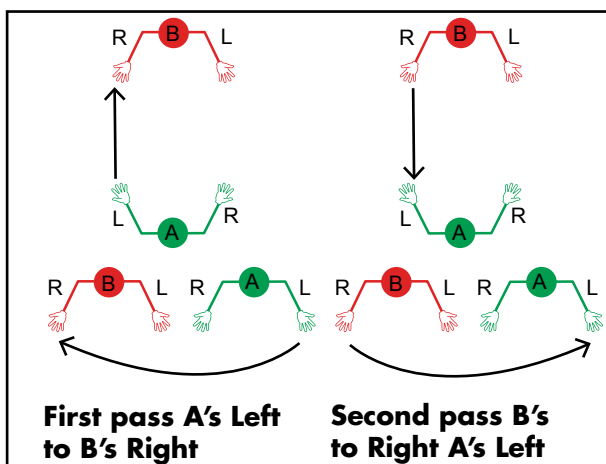
Below are the 4 possible hand circulations.



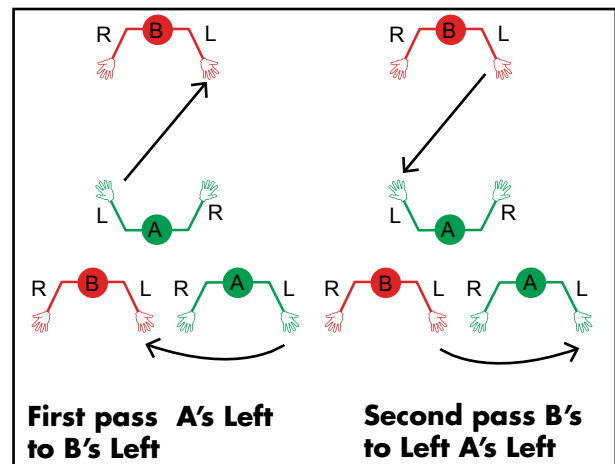
Circulation 1



Circulation 2



Circulation 3



Circulation 4

### 3) Bi

**Bi** in this context refers to twoness. Patterns with Bi passes are odd length patterns where the jugglers use both their left and right hands to pass and receive. In the face to face position they are juggled with one juggler crossing all passes and one juggler throwing all passing tramline straight.

Notice that in **Bi** patterns one passes out from the opposite hand to the one that received the previous incoming pass. This information is useful when starting complex patterns.

**Bi** passes are generated when:

an odd digit is transformed upwards within an odd period siteswap

or

an even digit is transformed downwards within an odd period siteswap.

On our lists Bi passes will be coloured Purple.

Notice that all Bi patterns have Decimal value of  $x.5$ . This is because in order to create a pass from an odd period siteswap, we are adding or subtracting a half of an odd digit which will always

finish by x.5.

## Examples Patterns

### 3 Objects

#### 2.5p 1 1

Three objects three count, a fun pattern which can be juggled with quarter spins sometimes called joe-passes. Joe pass is the stickman who demonstrates passing patterns in the Joe-Pass software, Wolfgang W who coded this wonderful software informs me that it was by accidental default that anything under a 3p gets thrown as a quarter spin.

### 5 Objects

#### 3 2.5p 2

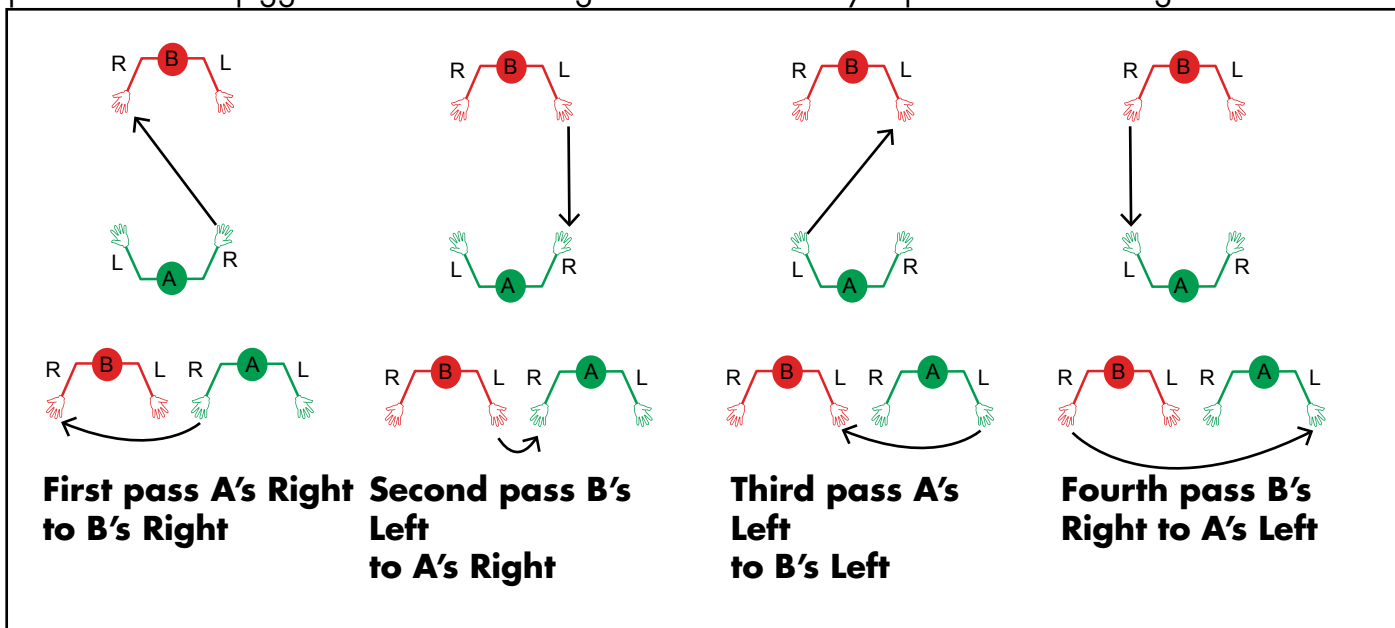
A good popcorn pattern. Also fun with joe-passes or pselfs as they Berlin group Take That Out call them.

### 7 Objects

#### < 4.5p 3 3 | 3 3 4.5p >

The standard 7 Object 3 count

There are just 2 ways of juggling these patterns. In the face to face position one juggler crosses his passes and one juggler throws them straight. The second way is just the mirror image of the first.



### 4) Instant-Bi

**Instant-Bi** patterns are extremely similar in feel to the Bi patterns. They are also odd length patterns where both hands pass and receive. They can also be juggled face to face with one juggler crossing and one juggler going straight. The difference is that the hand that catches the incoming pass is the hand that passes the next pass.

**Instant-Bi** passes are generated when:

an even digit is transformed upwards within an odd period siteswap.

an odd digit is transformed downwards within an odd period siteswap.

In our lists Instant-Bi patterns are colored green.

## Examples

3 Objects

3.5p 0 1

A charming simple pattern that feels like the two ball pattern 501.

5 Objects

3.5p 3 1

Can be learned as 3.5p22 progressively replacing the 22's with 31's.

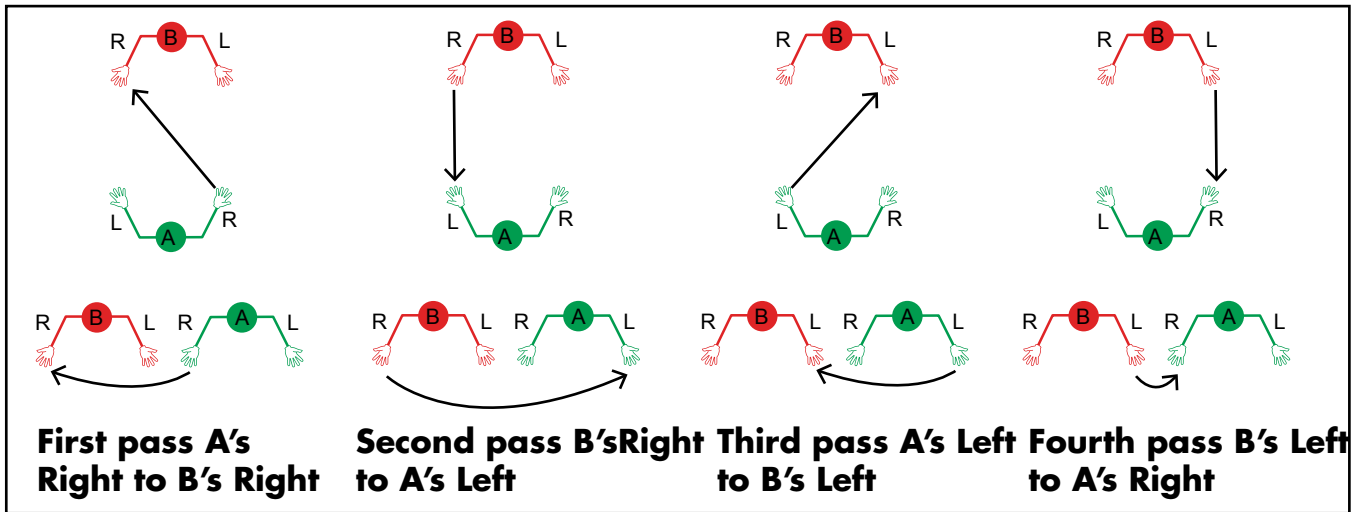
If juggled with clubs remember that it should be floaty singles or spiny doubles.

7 Objects

43.5p3

The popular French three count. As above the passes should float.

Below is an illustration of the hand circulation for Bi-Equi Passes. There are two circulations possible, the second being the mirror image of the first.



### Patterns with more than one pass

One can apply a Prechat transformation to more than one digit in the pattern and end up with patterns with more than one pass.

For example

1111 3p3p11 a 4 object 2 pass pattern

3333 5p5p33 an 8 object 2 pass pattern

This is where things start getting interesting and complicated.

It is worth remembering that our terminology relates just to the pass and not the pattern as a whole. So a pattern can have different kinds of passes.

Even period patterns can contain **Equihanded** passes and **Classic** passes.  
Odd period pattern can contain **Bi** passes and **Instant-Bi** passes.

Examples

When passing face to face it is useful to remember is that if a 3p (an odd pass) crosses then a 4p (an even pass) will be straight and vice versa. This is also true of decimal passes where one can disregard the decimal part. So if we decide to cross a 3.5p then a 4.5p will be straight.

Obviously patterns with more than one pass are more difficult to learn than patterns with just one pass. This where understanding the Prechac transformation can come in handy.

For example if we are trying to learn 42p11p a fun 4 object pattern. It might help to transform the pattern into the 5 object 1 pass pattern by doing a reverse Prechac on the second pass. Half the period is 2, so if we add 2 to the final pass we get:

42p13 which has more objects but is probably easier to juggle.

So to generalise: Starting from any siteswap with  $n$  objects you can create a pattern with  $2n+k$  objects where  $k$  is the amount of passes.

## Orbits of the objects in symmetric patterns

Another interesting application of the Prechac system is that when you transform a siteswap into a passing pattern the siteswap's object orbits are preserved.

Lets assume i want to create a 2 person patterns with a similar feel to the 2 ball 312. In 312 the "3" balls goes backwards and forwards "locked" in its orbit. If we augment the 1, by half the period, it becomes 2.5p. So we get the pattern 3 2.5p 2. Which feels very similar and stil has the locked 3s.

So for example the pattern 5241 has two orbits, It has one object going up and down as a 4 (0040) and 2 objects on a slow shower like pattern (5201). If we transform it into 54p41 a 7 object passing pattern the final pattern still has only 2 orbits, the 4s still bop up and down and there are now 5 objects doing 54p01.

## Synchronous

One can apply the Prechac transformation to synchronous patterns. Synchronous patterns convert to just 2 kinds of passes. **Classic passes** and **Equihanded passes**.

Digits which have an x transform to classic passes whilst digits without an x transform to equihanded passes.

Because of this there is not need to colour code the passes. Digits followed by and x keep the x



when converted.

## **Multiplex**

We can indeed apply the same set of rules to multiplex patterns as we do to all other patterns. These patterns have the same classification as the other 2 person patterns.

## **Synchronous Multiplex**

Synchronous multiplex patterns have the same classification as synchronous patterns.

## **MHN and polyrythms**

The Prechac transformation can indeed be applied to more complex notations...

So in fact in you are in the company of 5 jugglers who are good polyrhythmic multiplexers you can quickly and easily put together some fun pentagons.

## 3 jugglers

For 3 jugglers things get slightly more complex but more interesting. One now has a choice of adding or subtracting either 1 third of the period, or 2 thirds of the period in order to transform a throw into a pass. We call these different transformations levels of augmentation. So a siteswap to which one adds 1 third is single augmented and one to which one adds 2 thirds become double augmented.

For example if i start from the siteswap 33 which is period 2, i can transform it by adding a third:

33 we add 0.6 which is a third of 2, and we get: 3.6p<sub>3</sub> a 10 object 2 count triangle

or adding 2 thirds

33 we add 1.3 which is two thirds of the period and we get: 4.3p<sub>2</sub>3 an 11 Object 2 count triangle or dropback line

Notice that for clarity we have added a subscript of 2 after the p. This means that the pass has been augmented twice.

In terms of levels of augmentation subtraction does the opposite, if we deduct 1 third we end up with a double augmented pass and if we subtract 2 thirds we end up with a single augmented pass.

The difference between single augmented passes and double augmented passes, is the juggler passing order.

### **Pass direction versus juggler passing order.**

This is a very important concept to understand. The pass direction is from whom to whom the passes go, which juggler passes to which juggler; whilst the passing order is the order in which the jugglers pass.

The amount one adds to the pass affects the passing order of a pattern. So for example if we imagine three jugglers a,b,c juggling in a triangle where a throws to b, b throws to c and c throws to a. If the pass was generated adding 1/3rd of the period then the passing order will be the same as hand circulation, however if we add 2/3rds then the passing order will be the reverse of the hand circulation. We will see later that the same is true for more passers.

A good way of understanding this concept is to compare the classic 10 club and 11 club 2 count triangles. The 10 club triangle is 3 3.6p a single augmented pattern. The jugglers pass a to b, b to c, and c back to a. The passing order is the same. So if one starts the pattern cold a starts, followed by b and then by c. Simple and intuitive.

In the 11 club triangle however the passing order is the same, but if a starts, c will pass second.

## 3 person symmetric patterns

So once again we have 4 different kinds of passes. Each of these patterns can be either single augmented or double augmented.

Notice that we maintain the same colours coding system.

### 1) **Classic**

3 person **Classic** patterns are extremely similar in feel to 2 person patterns and indeed to classic patterns for any amount of jugglers and any degree of augmentation. They can be juggled all the rights to all the lefts.

In a triangle formation one can juggle them with all the passes going tramline straight. A number of these patterns were well known before siteswap notation.

**Classic** passes are generated when:

an odd digit is transformed upwards by  $1/3$ rd of the period within an even period siteswap.

an odd digit is transformed downwards by  $2/3$ rd of the period within an even period siteswap.

On our lists they will be coloured [blue](#).

Example patterns:

4 Objects

1 [1.6p](#)

A give the club to the next person shower. Both hands zapping.

7 Objects

[3.6p](#) 1

A pass followed by a zap two count.

3 [1.6p](#)

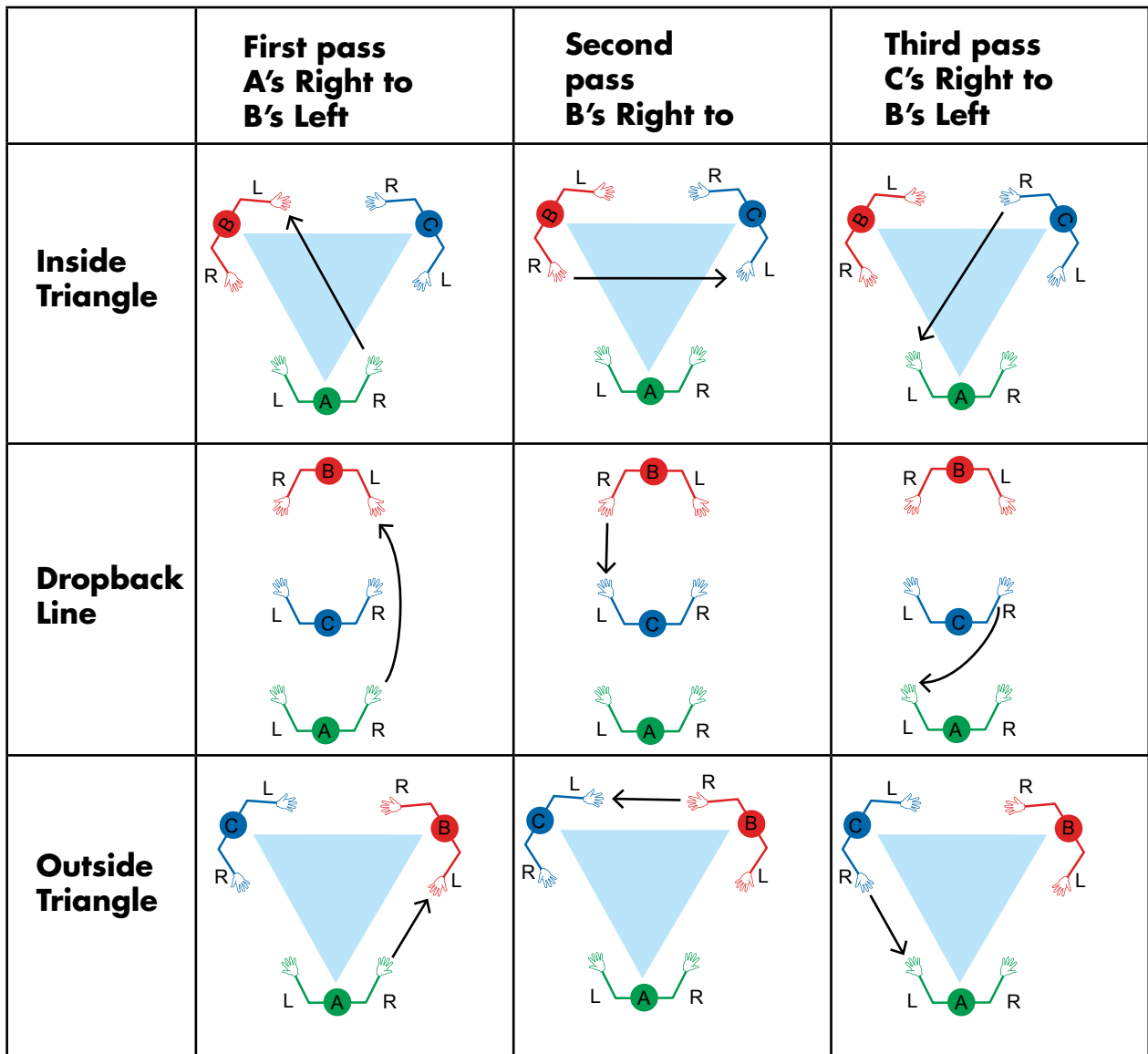
The opposite of the above. A self followed by a zap pass.

10 Objects

[33.6p](#)

The standart 10 Object 2 count. Notice from the [3.6p](#) that with clubs this works comfortably with both singles and doubles.

Below is an illustration of the most common way of juggling Classic patterns in three popular configurations.



The above is one of the six different hand circulations for classic patterns. Below is a table with all six of the circulations. Note that the second three circulations are reflections of the first three.

	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{C}$
1	R-L	R-L	R-L
2	R-L	R-R	L-L
3	R-R	L-R	L-L
4	L-R	L-R	L-R
5	L-R	L-L	R-R
6	L-L	R-L	R-R

## 2) Classic Augmented

Classic Augmented patterns are very similar to classic patterns. They can also be juggled with all the passes going from rights to lefts tramline straight. The main difference is that the passing direction and passing order are different.

Classic Augmented passes are generated when:

an odd digit is transformed upwards by 2/3rds of the period within an even period siteswap.

an odd digit is transformed downwards by 1/3rd of the period within an even period siteswap.

On our lists Classic Augmented patterns will be coloured blue and have a 2 as a subscript.

### Example patterns

5 Objects

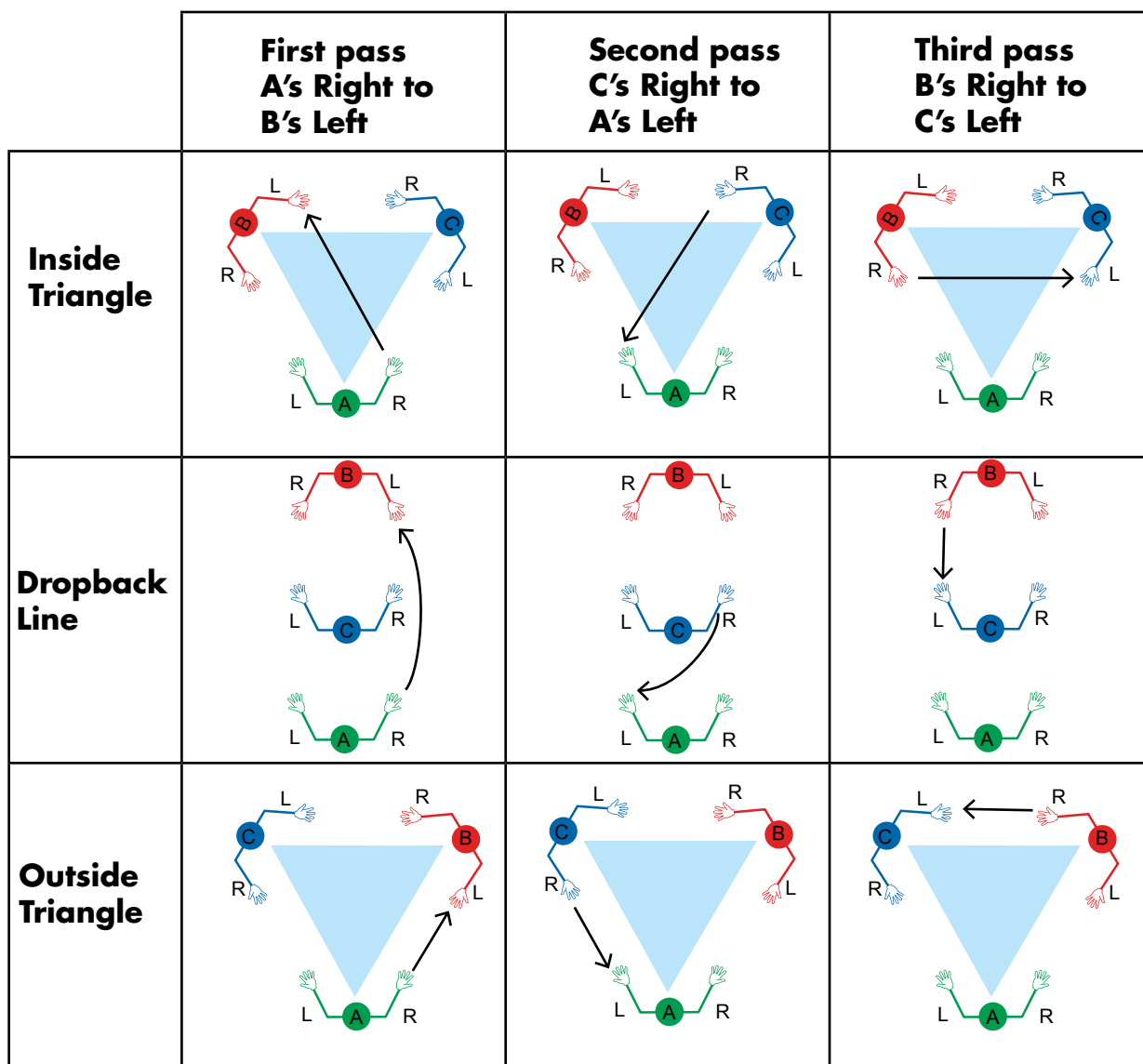
2.3p<sub>2</sub> 1 Fast passes across followed by a zap.

8 Objects

4.3p<sub>2</sub> 1 Big pass followed by a zap.

11 Objects

4.3p<sub>2</sub> 3 Is the standard 11 club triangle, usually done on floaty doubles.



The difference between the **Classic** and **Classic Augmented** diagrams is simply a switching of the second and third passes.

This is true of all the patterns which are augmented by 2/3rds and are written as p<sub>2</sub> in our lists.

Once in motion these patterns feel identical to the classic patterns. The main difference between

them is that the passing order and the passing direction are different. This means that if juggler A throws to Juggler B who throws to juggler C, then the order in which the jugglers begin throwing (and indeed continue throwing) will be A, C, B.

Below is a table with all six possible hand circulations for **Classic Augmented** passes.

	↻ <sup>A</sup>	↻ <sup>C</sup>	↻ <sup>B</sup>
1	R-L	R-L	R-L
2	R-L	R-R	L-L
3	R-R	L-R	L-L
4	L-R	L-R	L-R
5	L-R	L-L	R-R
6	L-L	R-L	R-R

### 3) Equihanded

The general rule for **equihanded** passes is that the hand that throws is the hand that receives a pass. This is true for equihanded patterns with any amounts of jugglers. So only one of the juggler's hands is involved in throwing and catching the pass. In a triangle one of the easiest ways of juggling these patterns is crossing all the passes.

In a dropback it's possible to juggle equihanded patterns with all the passes just on one side. See illustration

Equihanded passes are generated when:

an Even digit is transformed upwards by 1/3rd of the period, within an Even period siteswap.

an Even digit is transformed downwards by 2/3rds of the period within an Even period siteswap.

On our lists they will be coloured **red**.

#### Example patterns

4 Objects

**3.3p011**

4 Objects shared between three hands with a stolen zap-zap in between

7 Objects

**3.3p231** derived from 3.3p222 and similar to 3.3p312 useful for timing to throw the 2s.

10 Objects

**43.3p33** A nice popcorn pattern.

	⌚ <sup>A</sup>	⌚ <sup>C</sup>	⌚ <sup>B</sup>
1	R-R	R-R	R-R
2	R-R	R-L	L-R
3	R-L	L-R	R-R
4	R-L	L-L	L-R
5	L-L	L-L	L-L
6	L-L	L-R	R-L
7	L-R	R-L	L-L
8	L-R	R-R	R-L

Above is the table for the 8 possible circulations of Equihanded patterns. Note that the bottom 4 are reflections of the top 4.

Actually one can ignore this chart and just pick a hand for each juggler and know that the throws and catches will go between those hands.

#### 4) Equihanded Augmented

The Augmented Equihanded passes are very similar to the Equihanded passes, the only difference being the hand circulation. The passes still involve only one of the jugglers hands.

Equihanded Augmented passes are generated when an:

Even digit is transformed upwards by 2/3rds of the period within an Even period siteswap.

Even digit is transformed downwards by 1/3rd of the period within an Even period siteswap.

On our lists Classic Augmented patterns will be coloured red and have a 2 as a subscript.

Example patterns

5 Objects

2.7p<sub>112</sub>

11 Objects

43.3p A cheeky pattern with all three jugglers juggling two in one hand and passing the other five clubs between the three remaining hands.

	⌚ <sup>A</sup>	⌚ <sup>B</sup>	⌚ <sup>C</sup>
1	R-R	R-R	R-R
2	R-R	R-L	L-R
3	R-L	L-R	R-R
4	R-L	L-L	L-R
5	L-L	L-L	L-L
6	L-L	L-R	R-L
7	L-R	R-L	L-L
8	L-R	R-R	R-L

Above is the table for the eight possible circulations of Equihanded Augmented patterns. Note that the bottom four are reflections of the top four.

## 5) All Cross

The name All Cross comes from the most common way of juggling this pattern in a triangle in which all the passes cross. see illustration below. The pass leaves from from the hand that **didn't** receive the pass.

There is just one other way of juggling this kind of pass in a triangle: one juggler throwing all passes crossed and two jugglers throwing straight. Here too one passes from the hand that didn't receive the pass.

All cross passes are generated when:

an Odd digit is transformed upwards by 1/3rd of the period within an Odd period siteswap.

an Even digit is transformed downwards by 2/3rds of the period within an Odd period siteswap.

On our lists All cross patterns are **Purple**.

### Example patterns

4 Objects

2p11

Clubs in a triangle. An intriguing three count. Great to get used to 2ps. The pattern has a straightforward start.

J1 starts with 2 clubs one in each hand, J2 with one in the left and j3 with one in the right.

Each juggler throws towards the empty hand. Every club circulates. The pattern is uniorbitted.

7 Objects

4p12

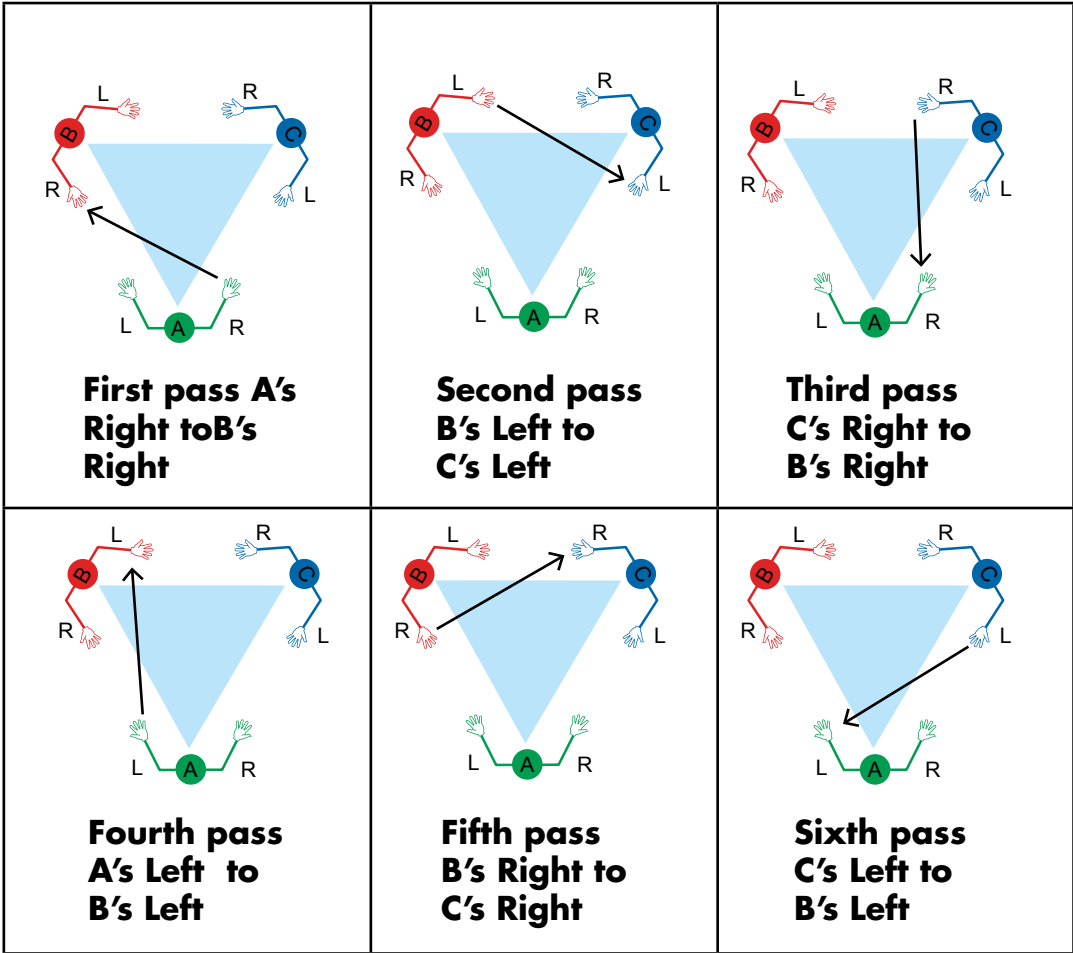
An easy three count pattern which can be used as practice for the pattern below.

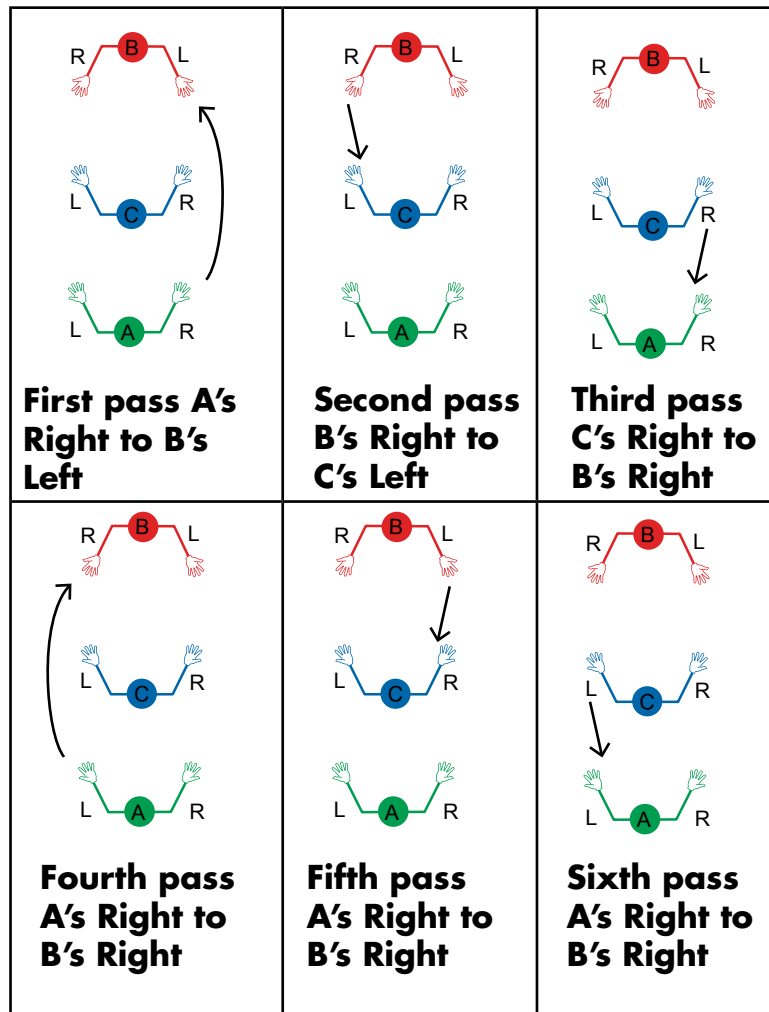
10 Objects

334p

Ten object three count. Lots of fun as a double spin triangle.







Above is the one of the ways of juggling all cross patterns in drop back line. Ironically the passes are all tramway so one could have called this category all straight.

1	R-R	L-L	R-R	L-L	R-R	L-L
2	R-R	L-R	L-R	L-L	R-L	R-L
3	R-L	R-L	R-R	L-R	L-R	L-L
4	R-L	R-R	L-R	L-R	L-L	R-L

Above are the four hand circulations for all cross patterns.

## 6) All Cross augmented

As with all the augmented patterns the hand circulation is the same but passing order is different. So these patterns can be juggled in a triangle with all the passes crossing. Here the hand that receives the pass is that hand that passes.

As in the All Cross you can juggle these patterns as all tramway dropback lines.

All Cross augmented passes are generated when:  
 an Even digit is transformed upwards by 2/3rds of the period within an Odd period siteswap.  
 an Odd digit is transformed downwards by 1/3rd of the period within an Odd period siteswap.

On our list all cross augmented are purple with a 2 subscript.

### Example patterns

5 Objects

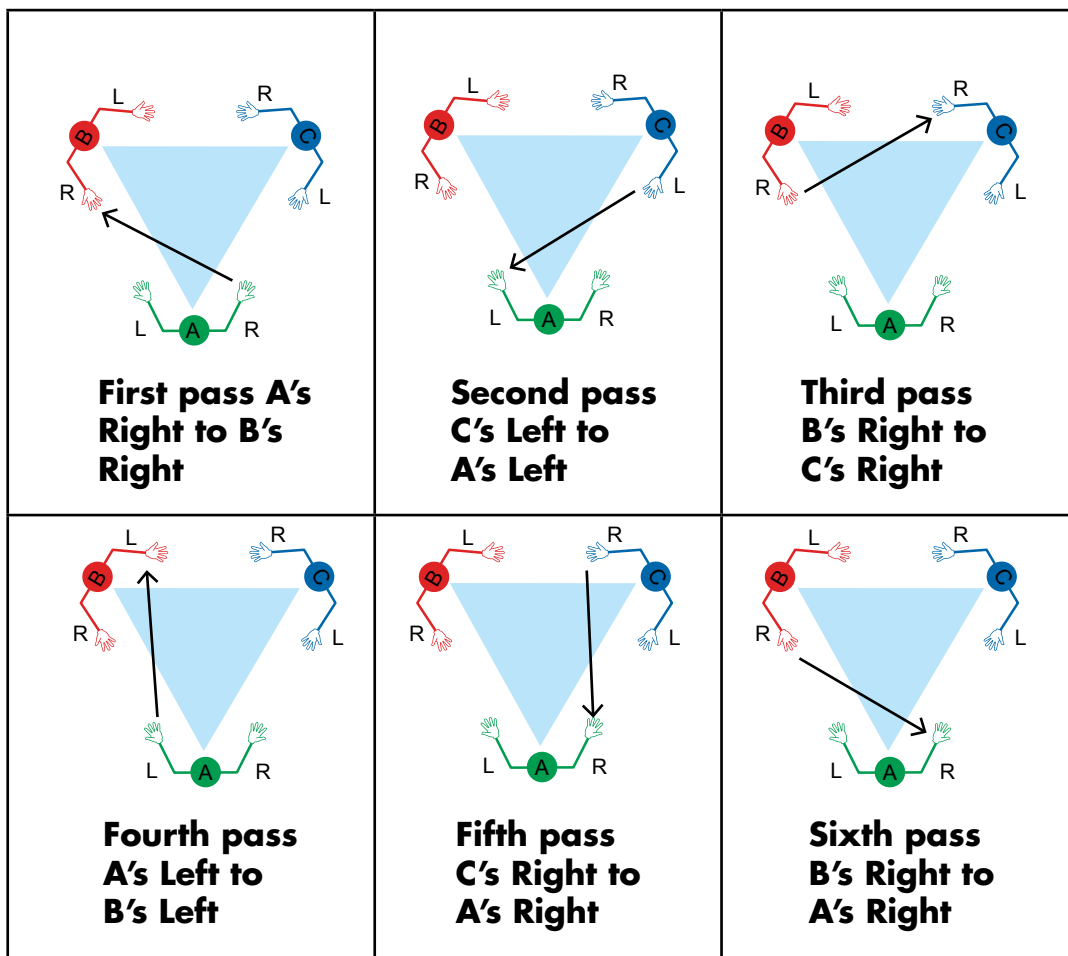
$4p_2 01$

8 Objects

$224p_2$

11 Objects

$344p_2$  A nice difficult popcorn.



	$\pi^A$	$\pi^C$	$\pi^B$	$\pi^A$	$\pi^C$	$\pi^B$
1	R-R	L-L	R-R	L-L	R-R	L-L
2	R-R	L-R	L-R	L-L	R-L	R-L
3	R-L	R-L	R-R	L-R	L-R	L-L
4	R-L	R-R	L-R	L-R	L-L	R-L

## 7) Tramway

Tramway passes get their name from the fact that one can juggle these patterns in a triangle with all passes going tramway straight. The hand that receives the pass is the hand that passes next.

Tramway passes are generated when:

an Even digit is transformed upwards by  $1/3$ rd of the period within an Odd period siteswap.

an Odd digit is transformed downwards by  $2/3$ rd of the period within an Odd period siteswap.

In our lists tramway patterns are **Green**.

### Example patterns

7 Objects

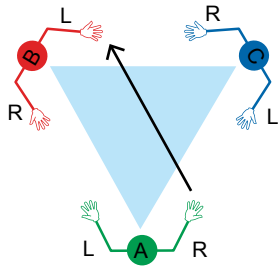
223p

A great way of learning how to do these patterns. Each juggler has two clubs and exchanges one of his clubs for the incoming pass.

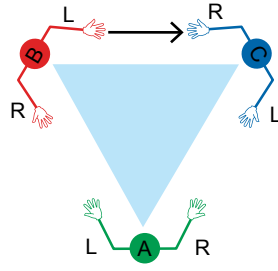
10 Objects

43p3

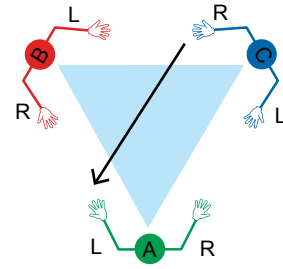
A charming but difficult three count similar to the popular two person French 3 count (43.5p3)



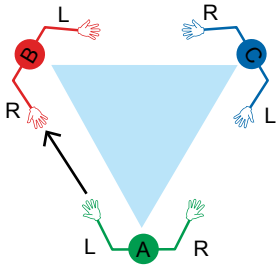
First pass  
A's Right to  
B's Left



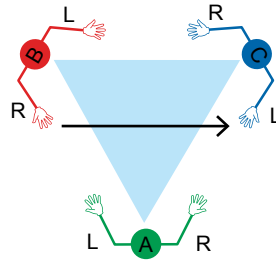
Second pass  
B's Left to  
C's Right



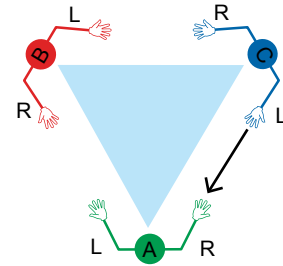
Third pass  
C's Right to  
A's Left



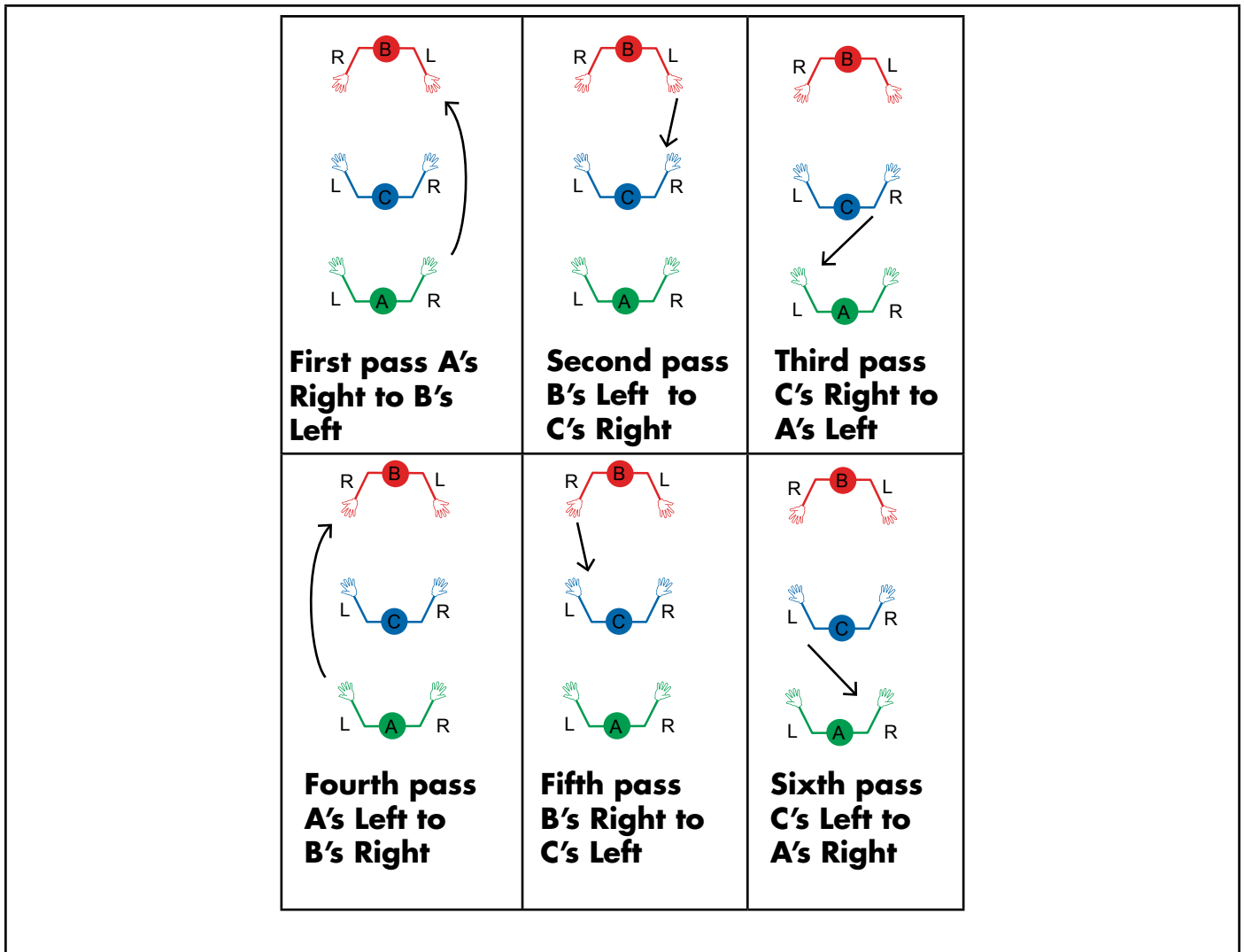
Fourth pass  
A's Left to  
C's Right



Fifth pass  
B's Right to  
A's Left



Sixth pass  
C's Left to  
B's Right



	$\sigma_A$	$\sigma_B$	$\sigma_C$	$\sigma_A$	$\sigma_B$	$\sigma_C$
1	R-L	L-R	R-L	L-R	R-L	L-R
2	R-R	R-R	R-L	L-L	L-L	L-R
3	R-L	L-L	L-L	L-R	R-R	R-R
4	R-R	R-L	L-L	L-L	L-R	R-R

## 8) Tramway Augmented

Tramway Augmented can be juggled like Tramway passes all passes going tramway straight. Once again the hand circulation and passing order are different. The passes leave from the hand that did not receive the previous incoming pass.

Tramway Augmented passes are generated when an Odd digit is transformed upwards by 2/3rd of the period within an Odd period siteswap.

Tramway Augmented passes are generated when an Even digit is transformed downwards by 1/3rd of the period within an Odd period siteswap.

In our list tramway augmented passes are green and have 2 Subscript.

## Example patterns

5 Objects

113p<sub>2</sub>

A zap zap pass pattern which is useful to get to grips with hand circulation

8 Objects

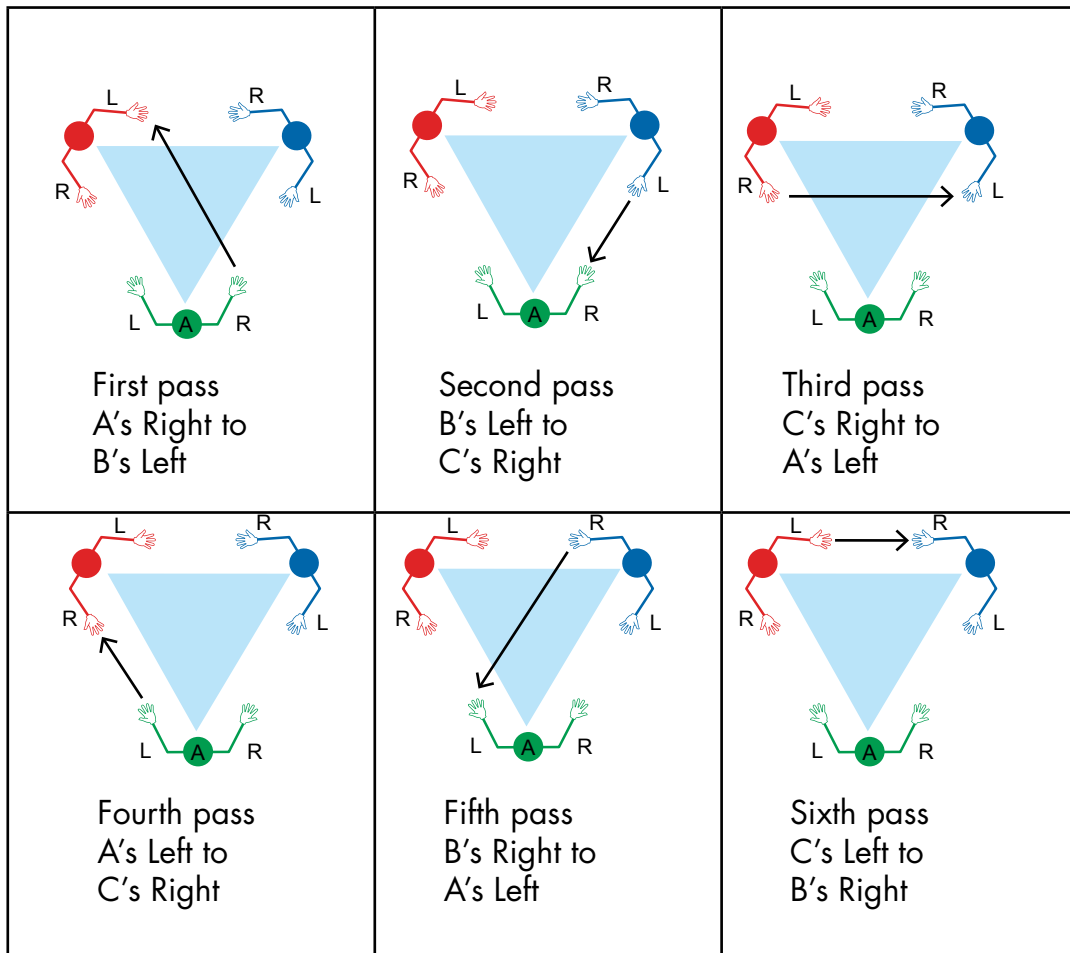
43p<sub>2</sub>1







A strange three count.

11 Objects

335p<sub>2</sub>

11 three count, a difficult three count with tripple passes.



						
1	R-L	L-R	R-L	L-R	R-L	L-R
2	R-R	R-R	R-L	L-L	L-L	L-R
3	R-L	L-L	L-L	L-R	R-R	R-R
4	R-R	R-L	L-L	L-L	L-R	R-R

## More passes

As we saw with the 2 person patterns, the categories refer to passes and not patterns. So patterns can contain more than one kind of patterns. As you can see from the colouring of the pattern lists,

no pattern contains more than two kinds of passes.

## Multidirectional patterns

As we saw above in patterns with 3 jugglers, the direction of the passes is dependant on the amount that one adds to the initial siteswap value. So we can imagine patterns with more than one pass where the passing order is different for each pass.

We will call these patterns multidirectional patterns. They are generated through a Prechac Transformation that adds different multiples of the period to different digits. In practical terms what this means is that each juggler passes to more than one other juggler.

For example:

531531 we remove  $1/3$  of the period from the first 5 and  $2/3$ rd from the second 5 and get  
 $3p_2 311 p_3 1$

This is a marvellous pattern for 6 objects that was first shown to me by Denis Paumier and Sylvain Garnavault where the 1ps circulate in one direction and the 3ps circulate in the opposite direction.

612612  $4p_2 122 p_1 2$   
711711  $5p_2 113 p_1 1$   
801801  $6p_2 014 p_1 01$

## Generalising the Notation to many jugglers

One can extend this to more jugglers. For  $m$  jugglers one has a choice of what to add to a siteswap digit to tranform it. One can add  $1/m$  of the period or  $2/m$  of the period up to  $m-1/m$  of the period. So for example if one is dealing with 5 jugglers one can add or substract  $1/5$ ,  $2/5$ ,  $3/5$  or indeed  $4/5$ .

In terms of the categorical equivalence adding  $x/m$  is the equivalent to substractin  $m-x/m$ . Lets look at a practical example: to make a 7 juggler pattern by adding  $6/7$ th will give the same category of pattern as substracting  $1/7$ . One can intuitively undestand why this is by remembering the siteswap rule, which says you can add the period to any of the digits in a siteswap and end up with another valid siteswap.

This also applies to multidirectional passing, so one can create patterns with more than one pass  
a nice 4 person distributive pattern.

717171  $2.5p_3 14p_2 15.5p_1 1$

## Many jugglers



As an example of the usefulness of this procedure lets generate the standard two counts for different amount of jugglers

Initial Siteswap	2 Jugglers	3 Jugglers	4 Jugglers	5 Jugglers	6 Jugglers	7 Jugglers
33	4p 3 7 Objects	3.6p 3 10 Objects	3.5p 3 13 Objects	3.4p 3 16 Objects	3.3p 3 19 Objects	3.3p 3 22 Objects
		4.3p <sub>2</sub> 3 11 Objects	4p <sub>2</sub> 3 14 Objects	3.8p <sub>2</sub> 3 17 Objects	3.6p <sub>2</sub> 3 20 Objects	3.6p <sub>2</sub> 3 23 Objects
			4.5p <sub>3</sub> 3 15 Objects	4.2p <sub>3</sub> 3 18 Objects	4p <sub>3</sub> 3 21 Objects	3.8p <sub>3</sub> 3 24 Objects
				4.6p <sub>4</sub> 3 19 Objects	4.3p <sub>4</sub> 3 22 Objects	4.1p <sub>4</sub> 3 25 Objects
					4.6p <sub>5</sub> 3 23 Objects	4.4p <sub>5</sub> 3 26 Objects
						4.7p <sub>6</sub> 3 27 Objects

## Multiples

A curious thing happens when we transform a siteswap into a 4 person passing pattern. There are 3 possible transformations, we can add 1/4, 2/4s or 3/4s. Adding 2/4s is the same as adding 1/2 so it follows that 2 person patterns can be done as 4 person patterns. Indeed they can be done as patterns for any even amount of jugglers.

More generally any patterns with y jugglers can done as patterns for multiples of y jugglers.

This is usefull to quickly transform known 2 or 3 person patterns into patterns for more people whilst keeping the familiarity of the known pattern.

So one change the 7 object 2 juggler 4 count patterns 5p333 into a 4 person y. Or the 3 person 10 object 3 count 4p33 into a 6 person spider....

So three person patterns can be done by six, nine or twelve jugglers.

## Glitches in the system

There is a grey area between staggered symmetric patterns and symmetric passing patterns. Lets take for example the siteswap 1111 and transform the first and third digits to end up with 3p13p1 this pattern is symmetric and not staggered symmetric in the sense that we have been talking about.

# 4 person symmetric patterns

## Even period patterns

Even period patterns follow an elegant simple progression as the amount of jugglers increases. They just generate classic passes and equihanded passes. The only thing that changes is the degrees of augmentation; which essentially means that the passing circulation stays the same but the passing order changes.

Single augmented patterns,  $p_1$ s have passing circulation and passing order the same. So can be done  $J_1 j_2 j_3 j_4$

Double augmented Have  $J_1$  and  $J_3$  starting first at the same time followed by  $J_2$  and  $J_4$ . Essentially one can think of these as 2 person patterns which have been doubled.

Triple augmented patterns:  $J_1 J_4 J_3 J_2$

## 1) Classic

The classic patterns are the most intuitive and resemble classic patterns for 2 and 3 jugglers. They can be done all the rights to all the lefts.

Illustration of box, Y and dropback lines.

Chart of the all the various permutations.

They can be augmented twice. So for example

$33.5p_1$  a 13 object 2 count

$34p_2$  a fourteen object 2 count that one could think of as 2 interlinked 7s.

$34.5p_3$  a fifteen object 2 count.

They are made by augmenting odd digits in even period siteswaps.

## 2) Equihanded

As with the classic patterns these patterns in essence stay the same. The hand that passes is the hand that receives.

Example patterns

They are made by augmenting even digits in even period siteswaps.

## Odd period patterns

### 1) Bi patterns single augmentation and tripple augmentation $p_3$

Bi patterns of single and tripple augmentation can be done with 3 jugglers crossing and 1 juggler throwing tramline straight or the opposite, 3 jugglers throwing tramline straight and 1 jugglers crossing.

They can't be juggled with all the jugglers doing the same thing.

Bi Patterns are generated by modifying odd digits within even period patterns.

	$\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{d}$	$\mathfrak{A}$	$\mathfrak{b}$	$\mathfrak{C}$	$\mathfrak{d}$	
1	R-L	R-L	R-L	R-R	L-R	L-R	L-R	L-L	Straight Straight Straight Cross
2	R-R	L-L	R-L	R-R	L-L	R-R	L-R	L-L	Cross Cross Straight Cross
3	R-R	L-R	L-L	R-R	L-L	R-L	R-R	L-L	Cross Straight Cross Cross
4	R-R	L-R	L-R	L-R	L-L	R-L	R-L	R-L	Cross Straight Straight Straight
5	R-R	L-L	R-R	L-R	L-L	R-R	L-L	R-L	Cross Cross Cross Straight
6	R-L	R-R	L-R	L-R	L-R	L-L	R-L	R-L	Straight Cross Straight Straight
7	R-L	R-L	R-R	L-R	L-R	L-R	L-L	R-L	Straight Straight Cross Straight
8	R-L	R-R	L-L	R-R	L-R	L-L	R-R	L-L	Straight Cross Cross Cross

The 8 possible hand circulations of Bi with single and tripple aumentation. Essentially when one juggles these patterns 3 people decide to do the same thing, either go straight or cross, and the fourth juggler will be forced to do the opposite.

### 2) Double augmentation $p_2$

Double augmented bis can be done all cross and all straight

### 3) Instant Bi single and tripple augmentation.

These patterns end up having the same circulation as the Bi patterns with the same augmentation. The difference is that the hand that recieves the pass is the hand that passes.

### 4) Instant Bi patterns Double augmentation $p_2$

These patterns can be juggled with 2 jugglers going straight and 2 jugglers crossing, or all jugglers going straight or all jugglers crossing.

## 5 and more people symmetric patterns

It seems unnecessary to go into every permutation of every amount of juggler. The following are just distilled bits of information

### 5 Person Bi Patterns

Bi P can be done all cross

Bi P<sub>2</sub> can be done all straight

Bi P<sub>3</sub> can be done all cross

Bi P<sub>4</sub> can be done all straight

### 5 Person Instant Bi Patterns

Bi P can be done all Straight

Bi P<sub>2</sub> can be done all cross

Bi P<sub>3</sub> can be done all Straight

Bi P<sub>4</sub> can be done all cross

### Passing order

Lets assume that the passing direction is j1 j2 j3 j4 j5

then if we augment by 2/5 the passing order will be:

j1 j4 j2 j5 j3

3/5

j1 j3 j5 j2 j4

4/5

j1 j5 j4 j3 j2

It seems unnecessary to go into every permutation of every amount of juggler.

## **Generalising to odd amounts of jugglers**

**the alteration between all cross and all straight on the levels of augmentation is true for all odd patterns**

### **6 jugglers**

Nothing here yet.

### **Feeds**

The subject of feeds is too vast to properly cover here. However there is an easy way to make balanced feeds. By balanced feeds we mean feeds where each feedee does the same job.

You can take any symmetric pattern and feed it with a siteswap equal to its length. You simply replace one of the siteswap values with a p for pass....

by then shifting the causal diagram in time we can i believe arrive at all symmetric feeds!...

### **Popcorn patterns**

All popcorn patterns are symmetric patterns. The reverse is not true. See article on popcorn patterns.

## Passing order generalised

If we take  $x$  jugglers who when augmented by  $1/\text{period}$  pass in the following order:

$j_1, j_2, j_3 \dots j_{x-1}, j_x$

Then if we augment a pass by  $y/x$ , where  $y < x$

then

Juggler number multiplied  $y \text{ Mod } x = \text{passing order number} - 1$

Juggler number multiplied  $y \text{ Mod } x = \text{passing order number} - 1$

For example if we have 7 jugglers and augment by  $2/7$ th and we want to know when the 3th juggler will throw. We do  $3 \times 2 \text{ mod } 7$  minus 1 Which gives us 5.

6 people

$p_1 \ j_1, j_2, j_3, j_4, j_5, j_6$

$p_2 \ j_1 \ j_4, j_2 \ j_5 \ j_3 \ j_6$  (jugglers start 2 at a time)

$p_3 \ j_1 \ j_3 \ j_5 \ j_2, j_4, j_6$  (jugglers start 3 at a time)

$p_5 \ j_1 \ j_4 \ j_3 \ j_6 \ j_2 \ j_5$

Check if the above calculations work with this and add circulation for 7!

## Nested Patterns













































